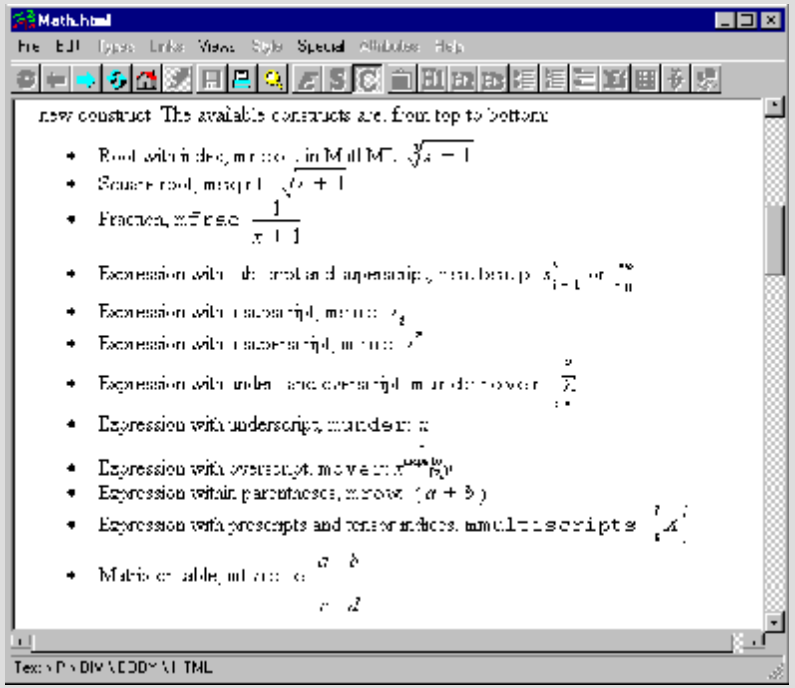
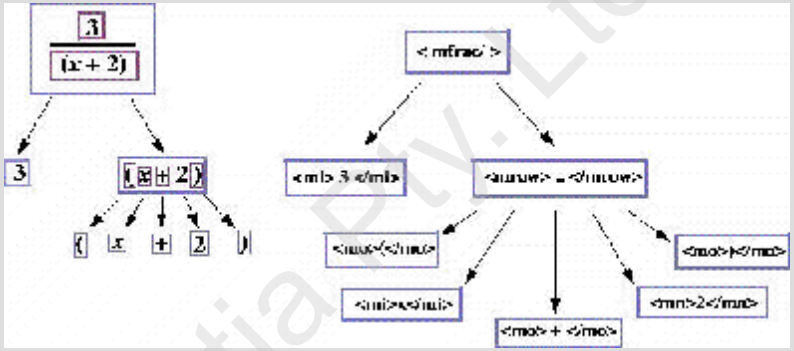
MathML

- **Mathematical Markup Language**
  - supporting equations was part of the *original* goal of the World-Wide Web
  - MathML is a W3C recommendation
    - *version 1.0.1 current, version 2.0 in progress*
  - rather verbose format
    - *all markup is contained within a <math> tag*
    - *uses prefix notation*
    - *editing tool support is important*
      - Amaya allows editing



- **A few points**
  - growing support
    - *IBM, Wolfram Research (Mathematica); Waterloo Maple (Maple V); Hewlett-Packard (EzMath plug-in); the American Mathematical Society; Design Science (MathType), Geometry Technologies (WebEQ), etc.*
  - it is natural to think about MathML expressions as tree structures



    - *thus they fit into XML's 'world' very well…*

Presentation vs. Content Markup

- **MathML can be used in two 'modes'**
  - presentation markup
    - *28 MathML presentation elements, with about 50 attributes*
    - *making marked-up data look nice*
  - content markup
    - *around 75 content markup elements, with about a dozen attributes*
    - *capturing the 'meaning' of the equation in a form suitable for automated processing*
  - <SEMANTIC> element
    - *allows two children: a presentation part and a content part*
      - content part could be a Java application, if desired

"…MathML has a dual purpose: to provide a standard for mathematics on the Web and to provide a mathematical notation, which encapsulates the content of the mathematics as much as possible. The equations can then be used where they sit in a document or can be pulled from a document to be used in an entirely different application."

MathML Gallery

- **Presentation markup**
  - supported by Amaya

```
<math>
  <msubsup>
    <mi>x</mi>
    <mrow>
      <mi>i</mi>
      <mo>+</mo>
      <mn>1</mn>
    </mrow>
    <mi>n</mi>
  </msubsup>
</math>
```

x_{i+1}^{n}

```
<math>
  <mrow>
    <msubsup>
      <mo>&Integral;</mo>
      <mn>0</mn>
      <mo>&infin;</mo>
    </msubsup>
    <mi></mi>
  </mrow>
</math>
```

$\int_0^\infty$

```
<mrow>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mrow>
        <mo>-</mo>
        <mi>b</mi>
      </mrow>
      <mo>&PlusMinus;</mo>
      <msqrt>
        <mrow>
          <msup>
            <mi>b</mi>
            <mn>2</mn>
          </msup>
          <mo>-</mo>
          <mrow>
            <mn>4</mn>
            <mo>&InvisibleTimes;</mo>
            <mi>a</mi>
            <mo>&InvisibleTimes;</mo>
            <mi>c</mi>
          </mrow>
        </mrow>
      </msqrt>
    </mrow>
    <mrow>
      <mn>2</mn>
      <mo>&InvisibleTimes;</mo>
      <mi>a</mi>
    </mrow>
  </mfrac>
</mrow>
```

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

```
<math>
  <mrow>
    <mi>y</mi>
    <mo>=</mo>
    <mi></mi>
    <mfrac href="http://www.w3.org/" xml:link="simple">
      <mn>1</mn>
      <msqrt>
        <mrow>
          <msup>
            <mi>x</mi>
            <mn>2</mn>
          </msup>
          <mo>+</mo>
          <mn>1</mn>
        </mrow>
      </msqrt>
    </mfrac>
  </mrow>
</math>
```

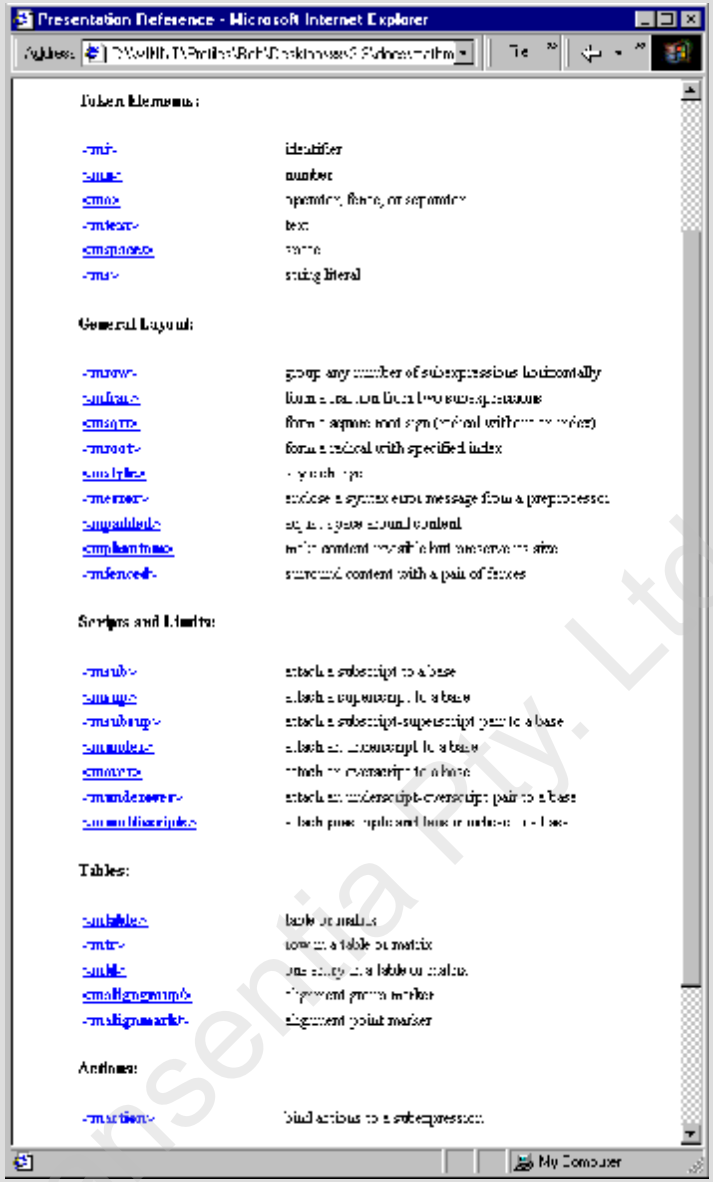
$y = \frac{1}{\sqrt{x^2+1}}$

This example also shows the use of an XLink

```
<math>
  <munderover>
    <mo>&Sum;</mo>
    <mrow>
      <mi>i</mi>
      <mo>=</mo>
      <mn>1</mn>
    </mrow>
    <mi>n</mi>
  </munderover>
</math>
```

$\sum_{i=1}^{n}$

Presentation Tags



Screendumps are from WebEQ documentation

• Content markup

$\sqrt{A^2 + B^2}$

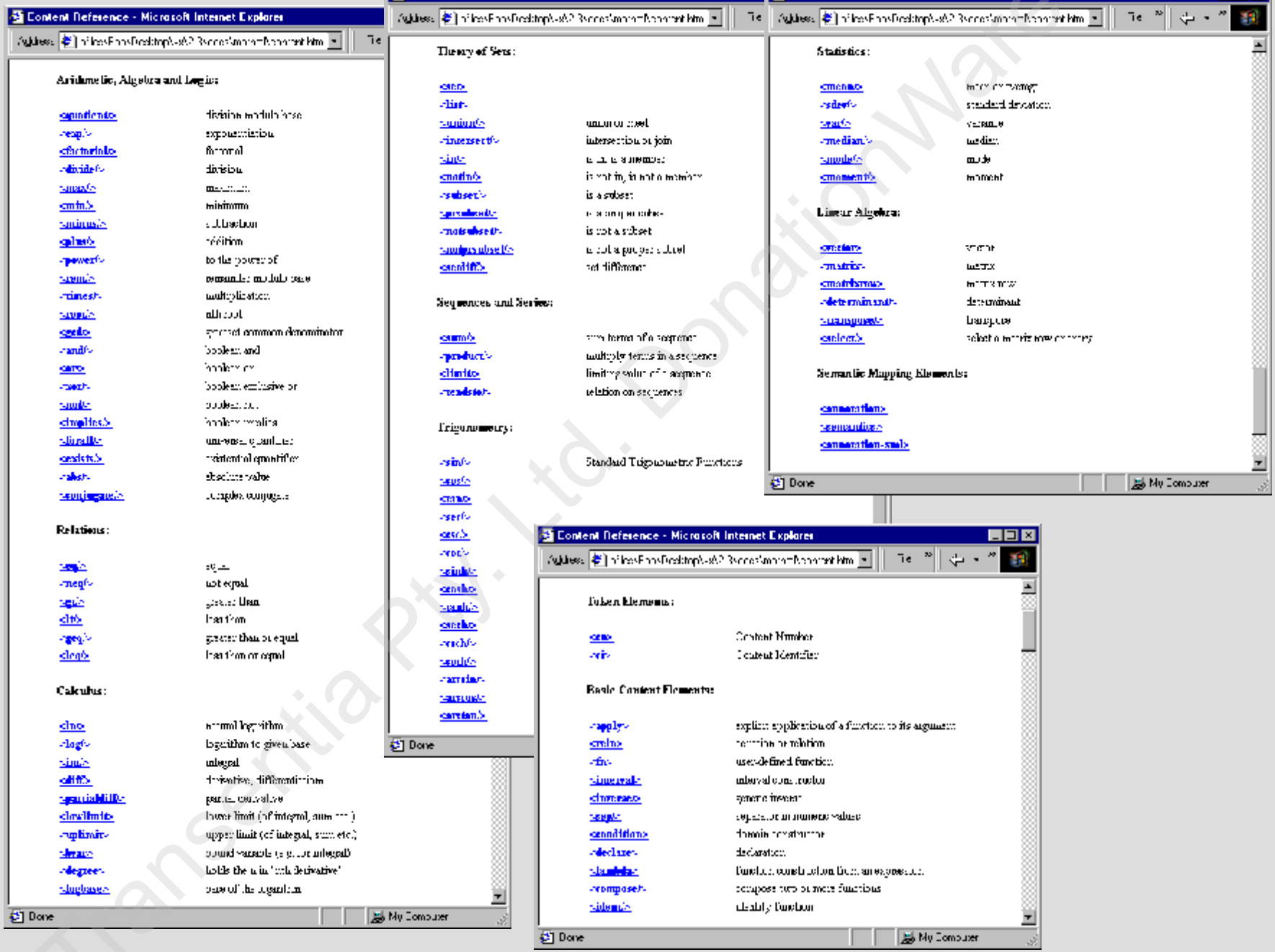
```
<math>
  <apply>
    <root/>
    <apply>
    <plus/>
    <apply>
      <power/>
      <ci>A</ci>
      <cn>2</cn>
    </apply>
    <apply>
      <power/>
      <ci>B</ci>
      <cn>2</cn>
    </apply>
    </apply>
  </apply>
</math>
```

$\{0\} = \{x \mid x \leq 0\} \cap \{x \mid x \geq 0\}$

```
<math>
  <reln>
    <eq/>
    <set>
      <cn>0</cn>
    </set>
    <apply>
      <intersect/>
      <set>
        <bvar><ci>x</ci></bvar>
        <condition>
          <reln>
            <le/>
            <ci>x</ci>
            <cn>0</cn>
          </reln>
        </condition>
      </set>
      <set>
        <bvar><ci>x</ci></bvar>
        <condition>
          <reln>
            <ge/>
            <ci>x</ci>
            <cn>0</cn>
          </reln>
        </condition>
      </set>
    </apply>
  </reln>
</math>
```

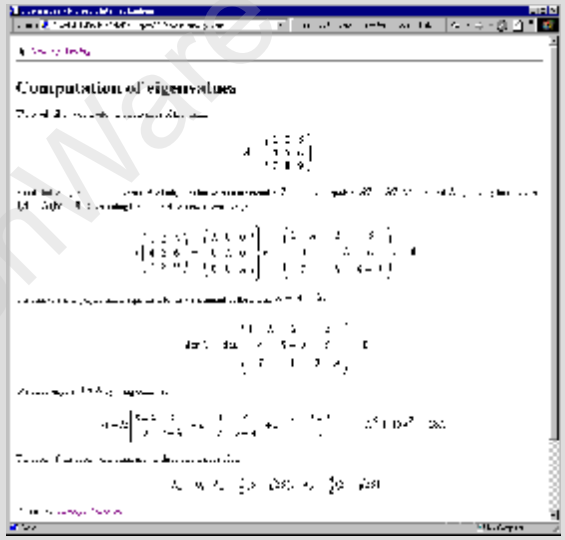
More Gallery

Content Tags



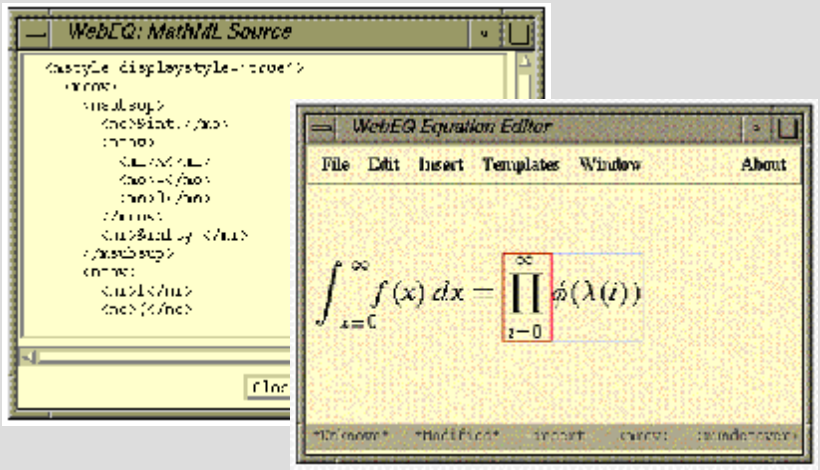
Tools: WebEQ

- **WebEQ**
  - an applet providing support in existing browsers
    - *also uses its own 'WebTex' language*




```
<APPLET CODEBASE="../../classes" ARCHIVE="webeq.zip"
        CODE="webeq.Main" WIDTH=560 HEIGHT=200>
<PARAM NAME=size VALUE=36>
<PARAM NAME=color VALUE="#ffffff">
<PARAM NAME=eq VALUE="\displaystyle{ \left( \sum_{k=1}^n a_k b_k \right)^2 \leq \left(
\sum_{k=1}^n a_k^2 \right) \left( \sum_{k=1}^n b_k^2 \right) } ">
</APPLET>
```

    - *has some nice 'smarts'*
  - also provides an editor



- **Free from W3C**
  - supports forms, tables and the most advanced features from HTML
    - *also support simple XLink, for example*
  - can create and edit complex mathematical expressions within Web pages
  - can style documents using Cascading Style Sheets
  - can publish HTML and XHTML documents
  - browsing and authoring are integrated
  - supports the editing and publishing of CSS style sheets
    - *can also easily link and unlink Cascading Style Sheets to documents. Also possible to selectively enable and disable them and see the rendering differences immediately.*

Tools: Mozilla

- **Aims to allow input and rendering of MathML**

MathML Basics - Mozilla {Build ID: 0000000000 }

With MathML, one can build sets such as (go on, right-click any of these equations to experiment the zoom) $\{0, 1, 2, 3, 4\}$ or $\left\{\left\lfloor\frac{a}{b}\right\rfloor \middle| a^2+b^2 \le 3\right\}$, write calculus $\frac{dy}{dx} = \frac{1}{y^2}$, form rather complicated expressions $\lim_{n\to N}\left(1+\frac{1}{n}\right)^n - e^N$, $k = \frac{\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2}{\left(1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right)^2}$, write vector equations $\boldsymbol{Y} = \boldsymbol{aX} + \boldsymbol{b}$, etc.

Notice how the mathematics appear in the main flow of text and respond as you resize the window. You can also make displayed equations, such as the following ones:

$$x \xrightarrow{\text{maps to}} y = f_n(x) = \left(1+\frac{1}{x^n}\right)^n$$

$$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{(b-a)^5}{4!5!}f^{(4)}(\eta), \qquad a \le \eta \le b$$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{otherwise} \end{cases}$$

You can also typeset 2D mathematical constructs such as matrices. The following example shows the i-th step of the multiplication of a matrix A by a vector x (notice how a_{i1}, ..., a_{in}, x_1 are on the same baseline, other alignments are possible):

$$i\text{-th row} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

In Mozilla, MathML runs inside the main browser. So it responds to other browser operations such as the zoom (try View -> Text Zoom), and you can do links $\underline{a^2}+\underline{b^2}=\underline{c^2}$, apply stylistic **effects** $\boldsymbol{a^2 + b^2 = c^2}$, or use color $a^2+b^2=c^2$ in very strange ways

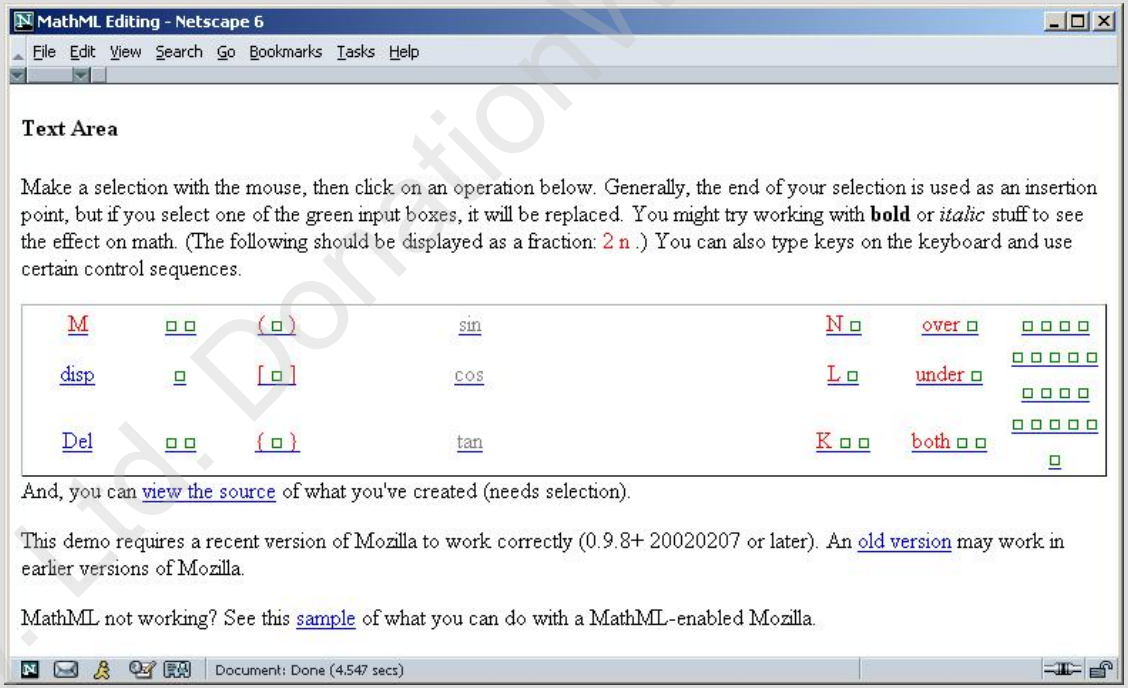
$$\frac{p(x)}{q(x)} = \frac{a_0+a_1x+a_2x^2+\dots+a_{n-1}x^{n-1}}{b_0+b_1x+b_2x^2+\dots+b_{n-1}x^{n-1}}$$

Document: Done (5.267 secs)

Tools: Javascripted MathML Editor

- http://www.newmexico.mackichan.com/MathML/mmled.xml
- Will eventually allow

```
<form>
  <object type="text/mathml">
    ...
  </object>
</form>
```




```
<a href="javascript:insertmatrix(2,2);"
   title="enter 2 by 2 matrix (CTRL 2)">
<mml:math>
  <mml:mtable>
    <mml:mtr>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
    </mml:mtr>
    <mml:mtr>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
    </mml:mtr>
  </mml:mtable>
</mml:math>
</a>
```

- **JavaScript MathML processor/ renderer**
  - http://www.pitt.edu/~jfcst24/mathmltest/legal.html
  - Also works with IE
    - *Early days for both…*