



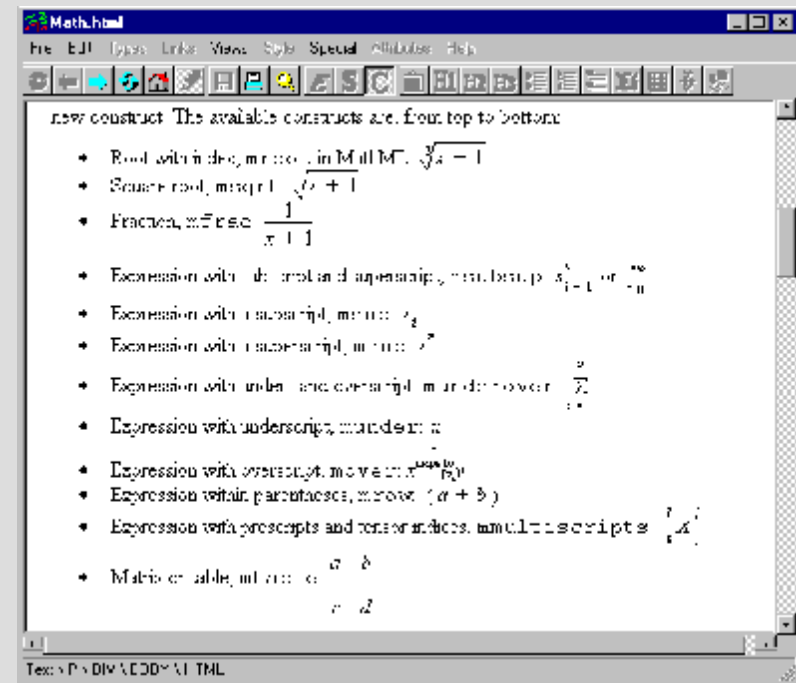
MathML

m1

Transentia Pty. Ltd. DonationWare

• Mathematical Markup Language

- supporting equations was part of the *original* goal of the World-Wide Web
- MathML is a W3C recommendation
 - *version 1.0.1 current, version 2.0 in progress*
- rather verbose format
 - *all markup is contained within a <math> tag*
 - *uses prefix notation*
 - *editing tool support is important*
 - **Amaya allows editing**

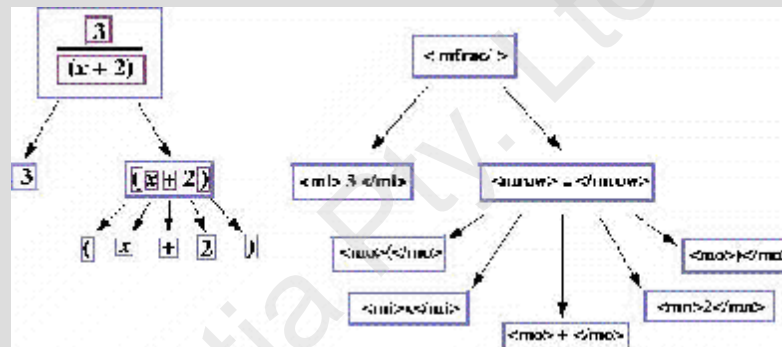


- **A few points**

- growing support

- *IBM, Wolfram Research (Mathematica); Waterloo Maple (Maple V); Hewlett-Packard (EzMath plug-in); the American Mathematical Society; Design Science (MathType), Geometry Technologies (WebEQ), etc.*

- it is natural to think about MathML expressions as tree structures



- *thus they fit into XML's 'world' very well...*

- MathML can be used in two 'modes'
 - presentation markup
 - 28 MathML presentation elements, with about 50 attributes
 - making marked-up data look nice
 - content markup
 - around 75 content markup elements, with about a dozen attributes
 - capturing the 'meaning' of the equation in a form suitable for automated processing
 - <SEMANTIC> element
 - allows two children: a presentation part and a content part
 - content part could be a Java application, if desired

"...MathML has a dual purpose: to provide a standard for mathematics on the Web and to provide a mathematical notation, which encapsulates the content of the mathematics as much as possible. The equations can then be used where they sit in a document or can be pulled from a document to be used in an entirely different application."

• Presentation markup

— supported by Amaya

```

<math>
  <msubsup>
    <mi>x</mi>
    <mrow>
      <mi>i</mi>
      <mo>+</mo>
      <mn>1</mn>
    </mrow>
    <mi>n</mi>
  </msubsup>
</math>

```

$$x_{i+1}^n$$

```

<math>
  <mrow>
    <msubsup>
      <mo>&Integral;</mo>
      <mn>0</mn>
      <mo>&infin;</mo>
    </msubsup>
    <mi></mi>
  </mrow>
</math>

```

$$\int_0^{\infty}$$

```

<math>
  <mrow>
    <mi>y</mi>
    <mo>=</mo>
    <mi></mi>
    <mfrac href="http://www.w3.org/" xml:link="simple">
      <mn>1</mn>
      <msqrt>
        <mrow>
          <msup>
            <mi>x</mi>
            <mn>2</mn>
          </msup>
          <mo>+</mo>
          <mn>1</mn>
        </mrow>
      </msqrt>
    </mfrac>
  </mrow>
</math>

```

$$y = \frac{1}{\sqrt{x^2 + 1}}$$

This example also shows the use of an XLink

```

<mrow>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mo>-</mo>
      <mi>b</mi>
    </mrow>
    <mo>&PlusMinus;</mo>
    <msqrt>
      <mrow>
        <msup>
          <mi>b</mi>
          <mn>2</mn>
        </msup>
        <mo>-</mo>
        <mrow>
          <mn>4</mn>
          <mo>&InvisibleTimes;</mo>
          <mi>a</mi>
          <mo>&InvisibleTimes;</mo>
          <mi>c</mi>
        </mrow>
      </mrow>
    </msqrt>
  </mfrac>
  <mrow>
    <mn>2</mn>
    <mo>&InvisibleTimes;</mo>
    <mi>a</mi>
  </mrow>
</mrow>

```

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```

<math>
  <munderover>
    <mo>&Sum;</mo>
    <mrow>
      <mi>i</mi>
      <mo>=</mo>
      <mn>1</mn>
    </mrow>
    <mi>n</mi>
  </munderover>
</math>

```

$$\sum_{i=1}^n$$

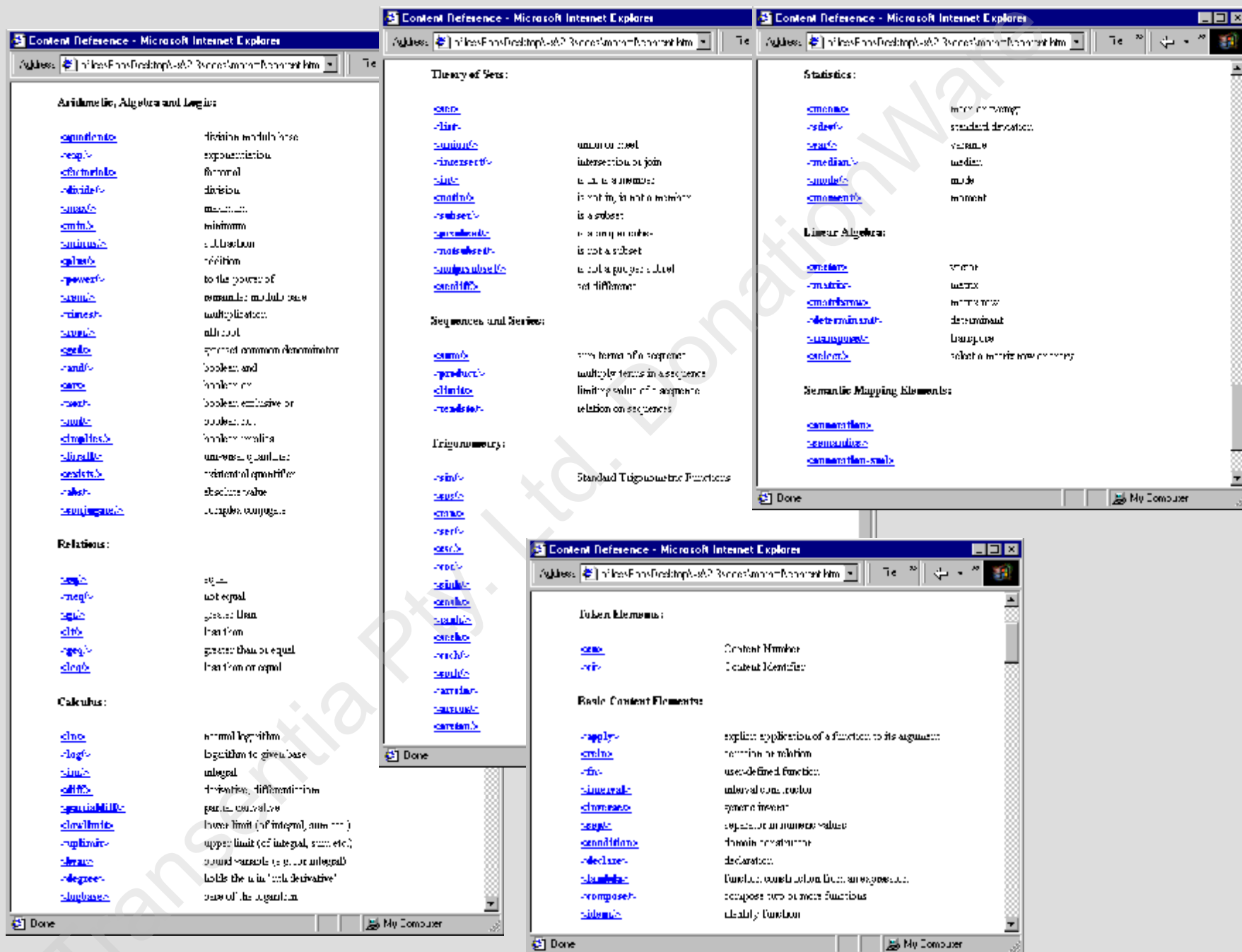
• Content markup

```
<math>
  <apply>
    <root/>
    <apply>
      <plus/>
      <apply>
        <power/>
        <ci>A</ci>
        <cn>2</cn>
      </apply>
      <apply>
        <power/>
        <ci>B</ci>
        <cn>2</cn>
      </apply>
    </apply>
  </math>
```

```
<math>
  <reln>
    <eq/>
    <set>
      <cn>0</cn>
    </set>
    <apply>
      <intersect/>
      <set>
        <bvar><ci>x</ci></bvar>
        <condition>
          <reln>
            <le/>
            <ci>x</ci>
            <cn>0</cn>
          </reln>
        </condition>
      </set>
    </set>
    <set>
      <bvar><ci>x</ci></bvar>
      <condition>
        <reln>
          <ge/>
          <ci>x</ci>
          <cn>0</cn>
        </reln>
      </condition>
    </set>
  </apply>
</reln>
</math>
```

More Gallery

Content Tags



The image displays three overlapping browser windows, each showing a different page of mathematical content tags. The windows are titled 'Content Reference - Microsoft Internet Explorer'.

Top Left Window: Arithmetic, Algebra and Logic

- [arithmetic](#): arithmetic
- [exp](#): exponential
- [form](#): formula
- [div](#): division
- [max](#): maximum
- [min](#): minimum
- [union](#): union
- [inter](#): intersection
- [power](#): to the power of
- [mult](#): multiplication
- [gcd](#): greatest common denominator
- [and](#): and
- [or](#): or
- [xor](#): exclusive or
- [implies](#): implies
- [forall](#): universal quantifier
- [exists](#): existential quantifier
- [value](#): value
- [output](#): output

Relations:

- [eq](#): equal
- [neq](#): not equal
- [less](#): less than
- [greater](#): greater than or equal
- [less_or_equal](#): less than or equal

Calculus:

- [log](#): logarithm
- [log_base](#): logarithm to given base
- [integrate](#): integral
- [deriv](#): derivative, differentiation
- [partial](#): partial derivative
- [limit](#): limit (of integral, sum, etc.)
- [upper_limit](#): upper limit (of integral, sum, etc.)
- [double_integrate](#): double integral (e.g. iterated integral)
- [derivative](#): holds the value 'derivative'
- [algorithm](#): name of the algorithm

Top Right Window: Theory of Sets

- [set](#): set
- [subset](#): subset
- [union](#): union
- [intersection](#): intersection
- [difference](#): difference
- [complement](#): complement
- [power_set](#): power set
- [cardinality](#): cardinality
- [finite](#): finite
- [infinite](#): infinite
- [countable](#): countable
- [uncountable](#): uncountable
- [disjoint](#): disjoint
- [subset_or_equal](#): subset or equal
- [superset_or_equal](#): superset or equal
- [proper_subset](#): proper subset
- [proper_superset](#): proper superset
- [equivalent](#): equivalent
- [isomorphic](#): isomorphic
- [homeomorphic](#): homeomorphic
- [isomorphic_to](#): isomorphic to
- [homeomorphic_to](#): homeomorphic to
- [isomorphic_or_homeomorphic_to](#): isomorphic or homeomorphic to
- [isomorphic_or_homeomorphic_to_or_equal](#): isomorphic or homeomorphic to or equal
- [isomorphic_or_homeomorphic_to_or_equal_or_less_than](#): isomorphic or homeomorphic to or equal or less than
- [isomorphic_or_homeomorphic_to_or_equal_or_less_than_or_equal](#): isomorphic or homeomorphic to or equal or less than or equal
- [isomorphic_or_homeomorphic_to_or_equal_or_less_than_or_equal_or_less_than_or_equal](#): isomorphic or homeomorphic to or equal or less than or equal or less than or equal

Statistics:

- [mean](#): mean
- [variance](#): variance
- [median](#): median
- [mode](#): mode
- [moment](#): moment

Linear Algebra:

- [matrix](#): matrix
- [vector](#): vector
- [determinant](#): determinant
- [rank](#): rank
- [trace](#): trace
- [eigenvalue](#): eigenvalue
- [eigenvector](#): eigenvector
- [singular_value](#): singular value
- [singular_vector](#): singular vector
- [singular_value_decomposition](#): singular value decomposition
- [singular_value_decomposition_to_rank_k_matrix](#): singular value decomposition to rank k matrix
- [singular_value_decomposition_to_rank_k_matrix_or_less_than](#): singular value decomposition to rank k matrix or less than
- [singular_value_decomposition_to_rank_k_matrix_or_less_than_or_equal](#): singular value decomposition to rank k matrix or less than or equal
- [singular_value_decomposition_to_rank_k_matrix_or_less_than_or_equal_or_less_than](#): singular value decomposition to rank k matrix or less than or equal or less than
- [singular_value_decomposition_to_rank_k_matrix_or_less_than_or_equal_or_less_than_or_equal](#): singular value decomposition to rank k matrix or less than or equal or less than or equal
- [singular_value_decomposition_to_rank_k_matrix_or_less_than_or_equal_or_less_than_or_equal_or_less_than_or_equal](#): singular value decomposition to rank k matrix or less than or equal or less than or equal or less than or equal

Bottom Window: Token Elements

- [token](#): token
- [token_id](#): token identifier

Basic Content Elements

- [apply](#): explicit application of a function to its argument
- [relation](#): relation
- [function](#): function
- [interval](#): interval
- [operator](#): operator
- [expression](#): expression
- [value](#): value
- [variable](#): variable
- [declaration](#): declaration
- [function_declaration](#): function declaration
- [composition](#): composition
- [identity](#): identity

• WebEQ

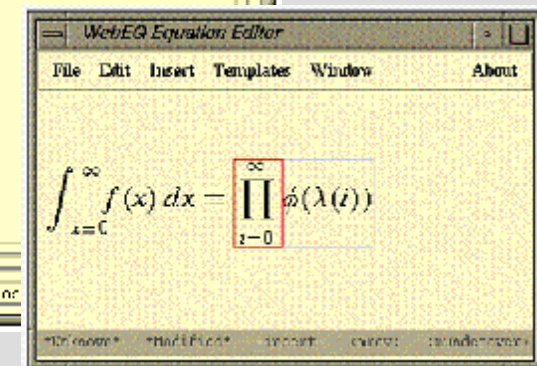
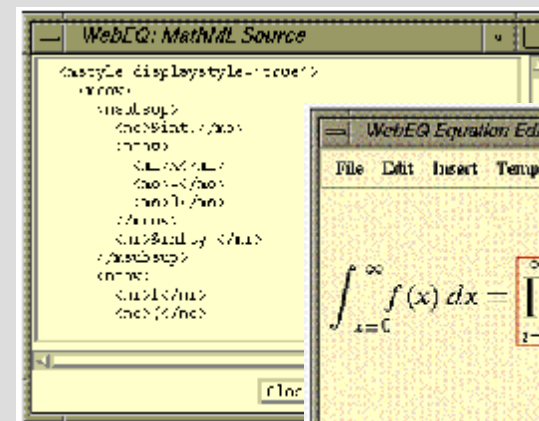
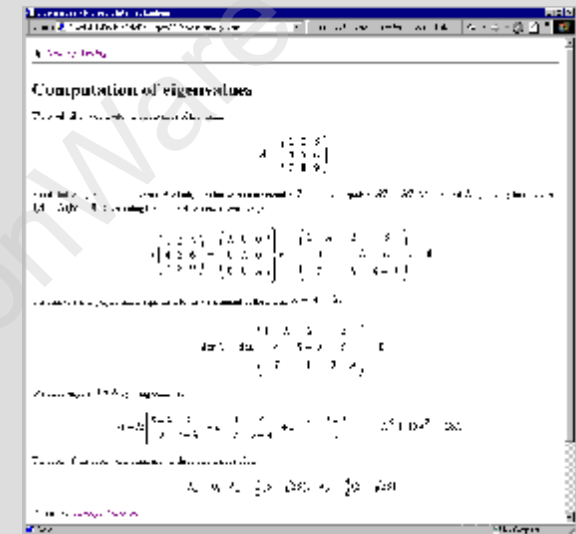
- an applet providing support in existing browsers

- *also uses its own 'WebTex' language*

```
<APPLET CODEBASE="../../../classes" ARCHIVE="webeq.zip"
        CODE="webeq.Main" WIDTH=560 HEIGHT=200>
<PARAM NAME=size VALUE=36>
<PARAM NAME=color VALUE="#ffffff">
<PARAM NAME=eq VALUE="\displaystyle{ \left( \sum_{k=1}^n a_k b_k \right)^2 \leq \left( \sum_{k=1}^n a_k^2 \right) \left( \sum_{k=1}^n b_k^2 \right) } ">
</APPLET>
```

- *has some nice 'smarts'*

- also provides an editor



- **Free from W3C**

- supports forms, tables and the most advanced features from HTML
 - *also support simple XLink, for example*
- can create and edit complex mathematical expressions within Web pages
- can style documents using Cascading Style Sheets
- can publish HTML and XHTML documents
- browsing and authoring are integrated
- supports the editing and publishing of CSS style sheets
 - *can also easily link and unlink Cascading Style Sheets to documents. Also possible to selectively enable and disable them and see the rendering differences immediately.*

• Aims to allow input and rendering of MathML

MathML Basics - Mozilla {Build ID: 0000000000 }

With MathML, one can build sets such as (go on, right-click any of these equations to experiment the zoom) $\{0, 1, 2, 3, 4\}$ or $\left\{\left[\frac{a}{b}\right] \mid a^2 + b^2 \leq 3\right\}$, write calculus $\frac{dy}{dx} = \frac{1}{y^2}$, form rather complicated expressions $\lim_{n \rightarrow N} \left(1 + \frac{1}{n}\right)^n - e^N$, $k = \frac{\frac{\partial^2 x}{\partial x^2} \frac{\partial^2 x}{\partial y^2} - \left(\frac{\partial^2 x}{\partial x \partial y}\right)^2}{\left(1 + \left(\frac{\partial x}{\partial x}\right)^2 + \left(\frac{\partial x}{\partial y}\right)^2\right)^2}$, write vector equations $Y = \mathbf{a}X + \mathbf{b}$, etc.

Notice how the mathematics appear in the main flow of text and respond as you resize the window. You can also make displayed equations, such as the following ones:

$$x \xrightarrow{\text{maps to}} y = f_n(x) = \left(1 + \frac{1}{x^n}\right)^n$$

$$\int_a^b f(x) dx = \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] - \frac{(b-a)^5}{4!5!} f^{(4)}(\eta), \quad a \leq \eta \leq b$$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{otherwise} \end{cases}$$

You can also typeset 2D mathematical constructs such as matrices. The following example shows the i -th step of the multiplication of a matrix A by a vector x (notice how $a_{i1}, \dots, a_{in}, x_1$ are on the same baseline, other alignments are possible):

$$i\text{-th row} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

In Mozilla, MathML runs inside the main browser. So it responds to other browser operations such as the zoom (try View -> Text Zoom), and you can do links $\underline{a^2 + b^2 = c^2}$, apply stylistic effects $\mathbf{a^2 + b^2 = c^2}$, or use color $\mathbf{a^2 + b^2 = c^2}$ in very strange ways

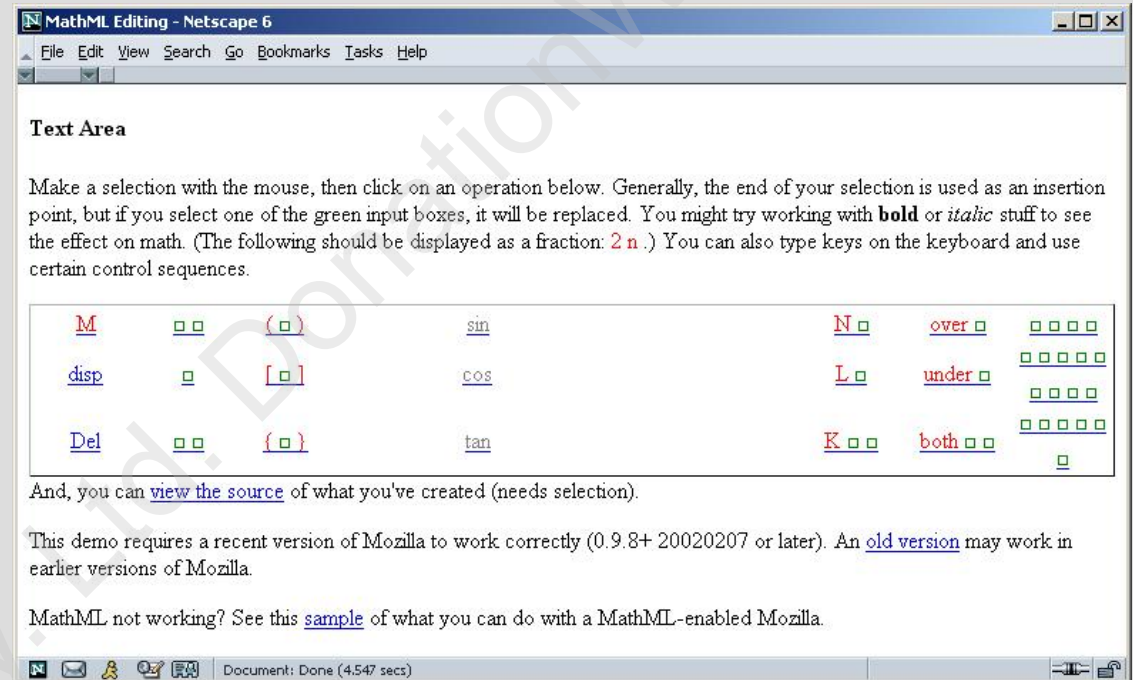
$$\frac{p(x)}{q(x)} = \frac{a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}}{b_0 + b_1 x + b_2 x^2 + \dots + b_{n-1} x^{n-1}}$$

Document: Done (5.267 secs)

- <http://www.newmexico.mackichan.com/MathML/mmlled.xml>
- Will eventually allow

```
<form>
  <object type="text/mathml">
    ...
  </object>
</form>
```

```
<a href="javascript:insertmatrix(2,2);"
  title="enter 2 by 2 matrix (CTRL 2)">
<mml:math>
  <mml:mtable>
    <mml:mtr>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
    </mml:mtr>
    <mml:mtr>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
      <mml:mtd> <mml:mi tempinput="true">&#x25A1;</mml:mi> </mml:mtd>
    </mml:mtr>
  </mml:mtable>
</mml:math>
</a>
```



- JavaScript MathML processor/ renderer
 - <http://www.pitt.edu/~jfcst24/mathmltest/legal.html>
 - Also works with IE
 - *Early days for both...*

